Areas of Planar Regions WORK SHEETS[1]

Activity 1.

- In Figure 1 select two points in the first quadrant at integer coordinates. Label the point with the larger x-value $\mathbf{P_1}$ and the other $\mathbf{P_2}$. Select $\mathbf{P_2}$ so that its y-coordinate is larger than that of $\mathbf{P_1}$. Connect the three points $\mathbf{P_0}$, which is at the origin, with the points $\mathbf{P_1}$ and $\mathbf{P_2}$ to form a triangle. (The integer coordinates are for convenience as are the restrictions on $\mathbf{P_1}$ and $\mathbf{P_2}$!)
- Drop a perpendicular from point $\mathbf{P_1}$ to the x-axis and label the point on the x-axis **R**. Drop a perpendicular from point $\mathbf{P_2}$ to the x-axis and label the point on the x-axis **Q**.
- Determine the area of $\Delta \mathbf{P_0P_2Q}$. ____________

  Determine the area of $\Delta \mathbf{P_0P_1R}$. ____________

  Determine the area of quadrilateral $\mathbf{QP_2P_1R}$. ____________
- Express the area of $\Delta \mathbf{P_0P_1\, P_2}$ as a linear combination of the areas from the previous steps. (Record this in symbols and numerically.

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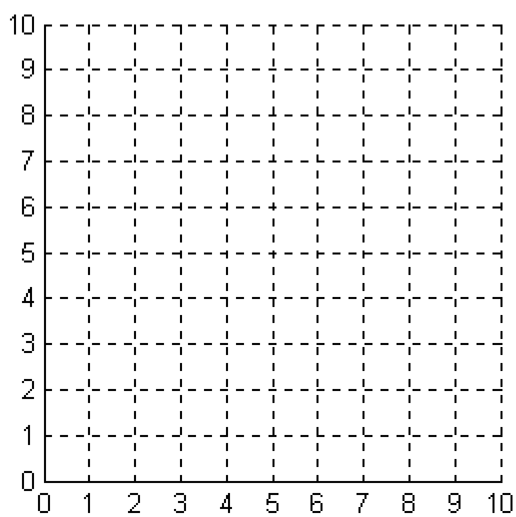

Figure 1.

Note: Area of a trapezoid = (1/2) height (sum of the lengths of the parallel sides).

[1] This material was developed by David R. Hill, Mathematics Department, Temple University and was used in Math 148 (Linear Algebra) in the Spring term of 2002. (hill@math.temple.edu)

Activity 2.

- In Figure 2 select two points in the first quadrant. Label the point with the larger x-value $\mathbf{P_1}$ and the other $\mathbf{P_2}$. Select $\mathbf{P_2}$ so that its y-coordinate is larger than that of $\mathbf{P_1}$. Denote the coordinates of $\mathbf{P_1}$ by $\mathbf{(x_1, y_1)}$ and those of $\mathbf{P_2}$ by $\mathbf{(x_2, y_2)}$. Connect the three points $\mathbf{P_0}$, which is at the origin, with the points $\mathbf{P_1}$ and $\mathbf{P_2}$ to form a triangle.
- Drop a perpendicular from point $\mathbf{P_1}$ to the x-axis and label the point on the x-axis **R**. Determine the coordinates of **R**. Drop a perpendicular from point $\mathbf{P_2}$ to the x-axis and label the point on the x-axis **Q**. Determine the coordinates of **Q**.
- Determine an expression for the area of $\Delta\mathbf{P_0P_2Q}$. ____________

  Determine an expression for the area of $\Delta\mathbf{P_0P_1R}$. ____________

  Determine an expression for the area of quadrilateral $\mathbf{QP_2P_1R}$. ____________
- Show that a formula for the area of $\Delta\mathbf{P_0P_1\,P_2}$. is given by $(1/2)[x_1y_2 - x_2y_1]$. Show your work.

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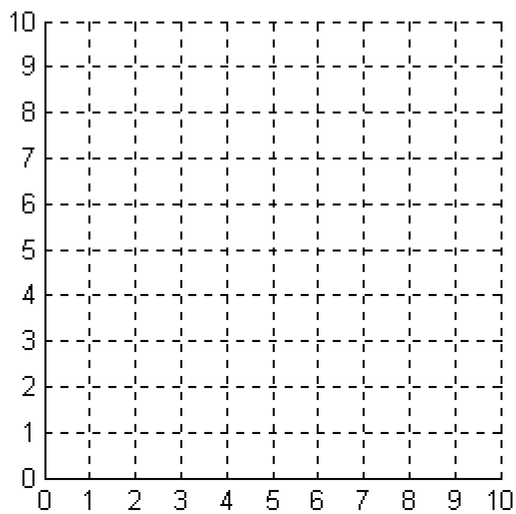

Figure 2.

- Check your 'formula' for the area of $\Delta\mathbf{P_0P_1\,P_2}$ using the following information.
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  1. If $\mathbf{P_1}$ has coordinates (5,2) and $\mathbf{P_2}$ has coordinates (3, 5), then the area of $\Delta\mathbf{P_0P_1\,P_2}$ is 19/2 square units.
  2. If $\mathbf{P_1}$ has coordinates (8,1) and $\mathbf{P_2}$ has coordinates (1, 7), then the area of $\Delta\mathbf{P_0P_1\,P_2}$ is 27.5 square units.
  3. If $\mathbf{P_1}$ has coordinates (7.5,2) and $\mathbf{P_2}$ has coordinates (0, 3), then the area of $\Delta\mathbf{P_0P_1\,P_2}$ is 11.25 square units.

- If you did <u>not</u> get the areas given in the previous 3 'tests', check your work for the formula you developed for the area of $\Delta\mathbf{P_0P_1\,P_2}$. Use the 3 'tests' to recheck your expression.

Activity 3. (Plot points in Figure 3 when you need a geometric view of the triangles.)

- Use the formula you developed for the area of $\Delta \mathbf{P_0 P_1 P_2}$ for each of the following cases:
  1. $\mathbf{P_1}$ = (2, 5) and $\mathbf{P_2}$ = (6, 7); area = ________________
  2. $\mathbf{P_1}$ = (5, 9) and $\mathbf{P_2}$ = (3, 2); area = ________________

  Describe any difficulties encountered interpreting your answers.

  How should you revise your formula developed previously to obtain a meaningful answer for the area in these cases?

  Revised formula for the area of $\Delta \mathbf{P_0 P_1 P_2}$ when $\mathbf{P_1} = \mathbf{(x_1, y_1)}$ and $\mathbf{P_2} = \mathbf{(x_2, y_2)}$ is

  ____

- Check that your revised formula makes sense for each of the following cases (draw a diagram of the triangle in Figure 3 as needed):
  1. $\mathbf{P_1}$ = (-2, 5) and $\mathbf{P_2}$ = (0, 7); area = ________________
  2. $\mathbf{P_1}$ = (3, -2) and $\mathbf{P_2}$ = (-5, -9); area = ________________
  3. $\mathbf{P_1}$ = (-3, 5) and $\mathbf{P_2}$ = (6, -7); area = ________________

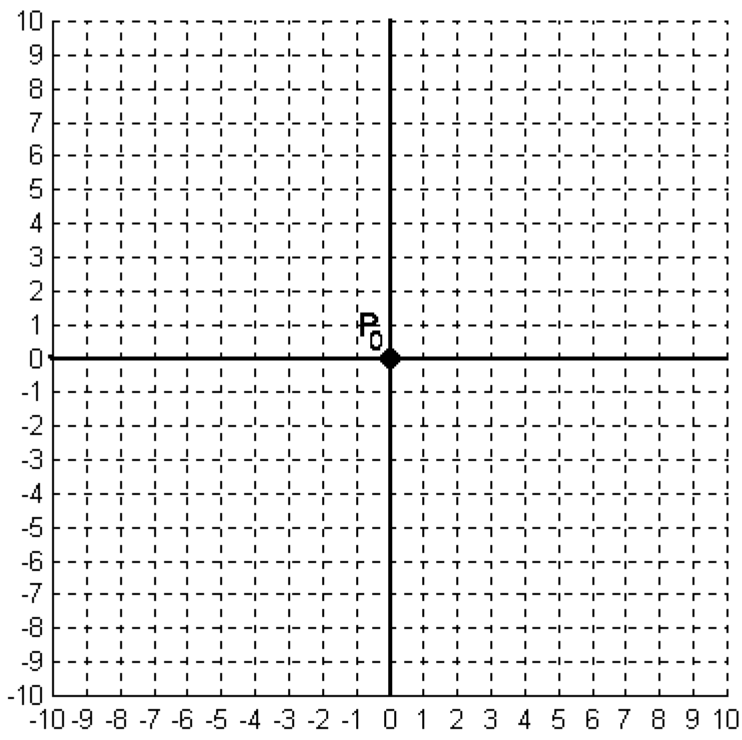

Figure 3.

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- In the checks above we specified points outside the first quadrant and did not name the points based on a clockwise orientation of the points; that is, the points were not named based on the size of x-coordinate as done previously. In fact, the "revised" formula above works for any pair of points $\mathbf{P_1}$ and $\mathbf{P_2}$ to compute the area of $\Delta\mathbf{P_0P_1P_2}$, where $\mathbf{P_0}$ is the origin.

  Prove that the result is valid for the triangle in Figure 4. Show your work.

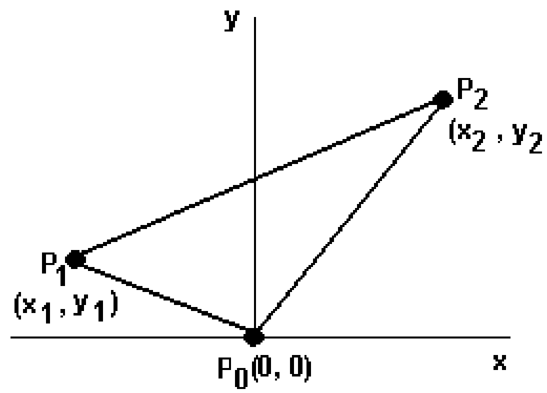

Figure 4.

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Activity 4.

- Here we consider ∆**ABC** depicted in Figure 5. The triangle is shown in the first quadrant for convenience. Use the ideas in the previous activities to develop a formula for the area of ∆**ABC**. (Use clockwise orientation for triangle names and coordinate information in area formulas.)

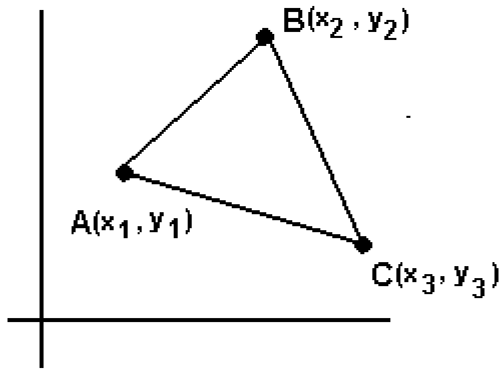

Figure 5.

Show your work below.

Formula for the area of ∆**ABC** = ________________________________

- Check your formula on the information given below.

1. For **A** = (7, 5), **B** = (5, 7), **C** = (2, 0), area = 10 square units.
2. For **A** = (8, 2), **B** = (6, 3), **C** = (0, 6), area = 0 square units.
3. For **A** = (2,0), **B** = (5, 7), **C** = (7, 5), area = 10 square units.
4. For **A** = (-2,1), **B** = (5, -3), **C** = (6, 5), area = 30 square units.
5. For **A** = (6, 5), **B** = (5, -3), **C** = (-2, 1), area = 30 square units.

If your formula needs to be revised do so below:

Revised formula for the area of ∆**ABC** = ____________________________

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Activity 5.

The formula for the area of a triangle generalizes so that we can compute the area of a closed polygon. Label the vertices of the polygon **P** in either a clockwise or counter-clockwise orientation as $P_1(x_1,y_1)$, $P_2(x_2,y_2)$,…. $P_n(x_n, y_n)$. So that we have a closed polygon we define $P_{n+1}(x_{n+1}, y_{n+1}) = P_1(x_1,y_1)$. Form a conjecture for an expression for the area of the polygon by generalizing the revised formula for the area of Δ**ABC** from Activity 4.

Conjecture: area of polygon **P** = ________________________________

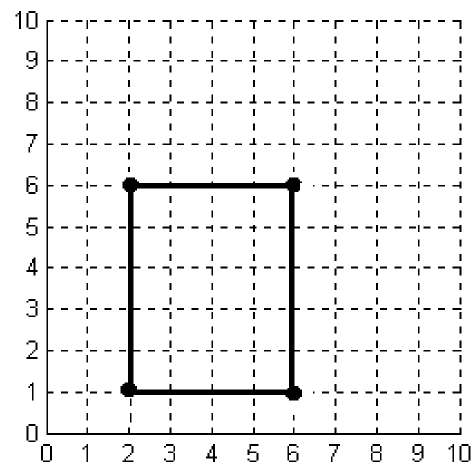
Check your conjecture using the following sets of information. (All vertices have integer coordinates.)

Rectangle in Figure 6 has area 20 square units.

Trapezoid in Figure 7 has area 25 square units.

Pentagon in Figure 8 has area 89.5 square units.

In these three examples you were given the area. Explain how to compute the area using only the "triangle" formula developed previously.



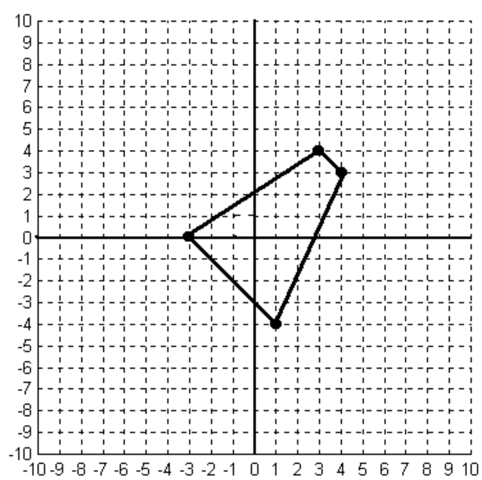


Figure 7.

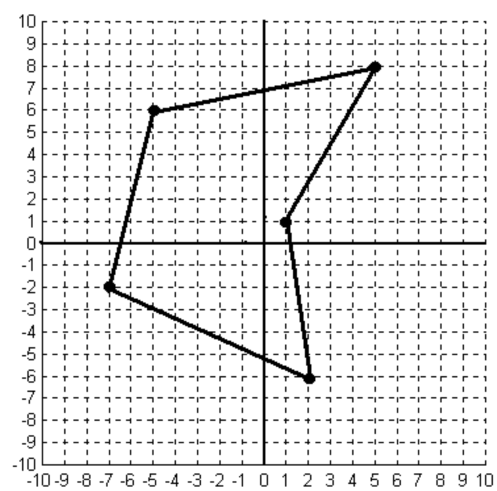


Figure 8.

Revise your conjecture if necessary.

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Activity 6.

Develop a technique for approximating each of the nonpolygonal regions shown in Figures 9 and 10.

- Explain your technique and give a rationale as to why it works.

- Relate the expression for the area of a polygon in Activity 5 to your technique.

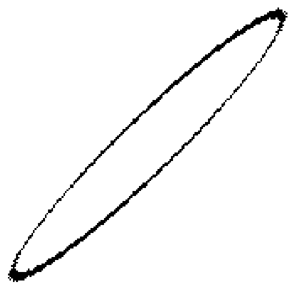

Figure 9.

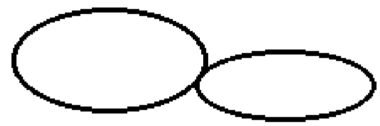

Figure 10.

- Apply your technique to accurately estimate the area of the region shown in Figure 11.

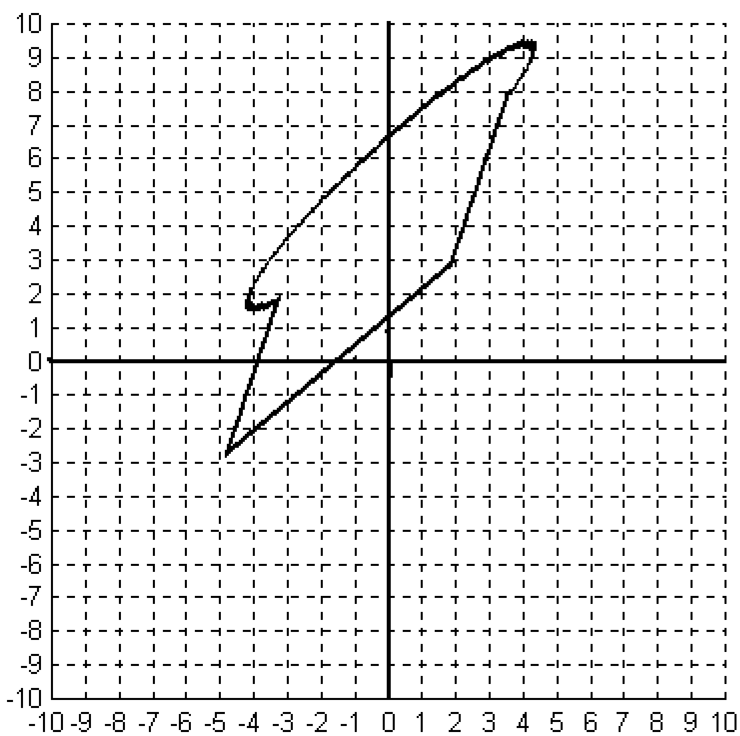

Figure 11.

Area estimate = ________________

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