Sample Numerical Experiments[1]

The assignment uses Flash programs that are available on the web. There are 3 programs:

1. Approximate area under a curve by selecting a partition to construct trapezoids that need not have the same height. See Figure 1.
2. Approximate area under a curve by selecting a partition to implement Simpson's Rule which uses parabolic arcs which need not have points evenly spaced. See Figure 2.

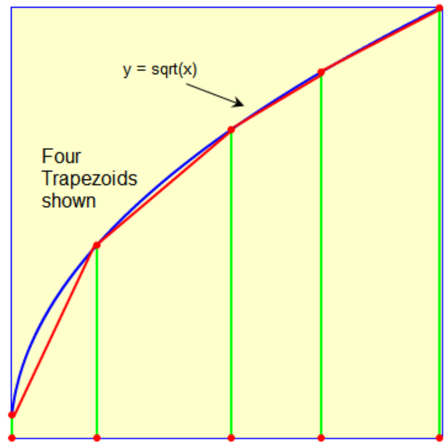


Figure 1.

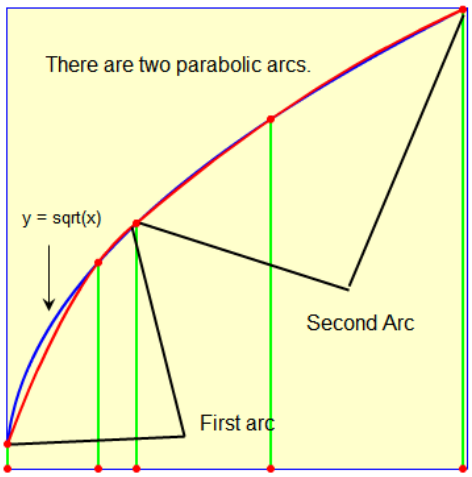


Figure 2.

3. Approximate the length of a curve, this is called arc length. Here we select points on the graph of the curve and connect them with straight line segments and add up the length of the segments to get an approximation. See Figure 3.

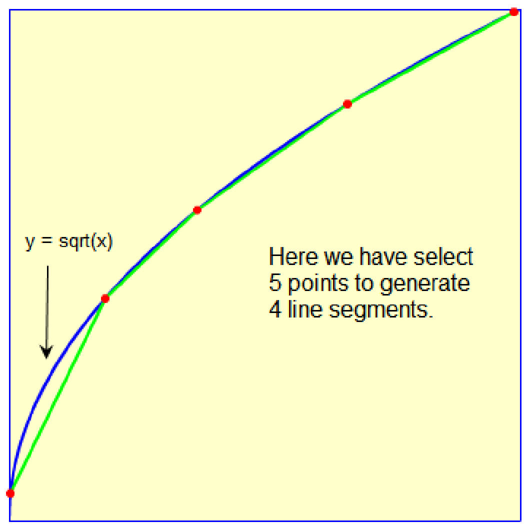
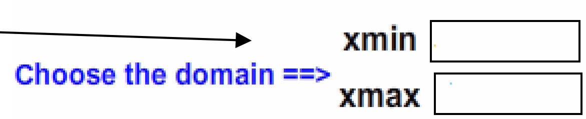


Figure 3.

For each routine you must do the following:

- Enter your name in the box at the top left of the screen, and then click the start button.

- Next type the values for the domain in the boxes displayed
  In order to use π, type **pi** or for 2π use **2*pi**, etc.



- Next enter an expression for the function you want to use in the box at the lower left above the button that says.

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There are certain rules for typing functions. Clicking the SYNTAX button displays the information shown below.

Syntax and Instructions

Use the f(x) to enter your function. The independent variable must be x. Use the standard syntax. For example:

x^3+2.7*sin(pi*x)+e^(-x+1)

Always use parenthesis around functions' arguments, and * to denote multiplication. / denotes division, ^ exponentiation.

Here is the list of functions and constants that the mathlet recognizes.

--- Trigonometric functions and inverse trigonometric functions:
sin( ), cos( ), tan( ), asin( ), acos( ), atan( ). For example: tan(pi*x/3)+acos(x)
--- ln( ) stands for the natural logarithm, e for the natural base. For example: e^(2-x)*ln(x^2+1)
Moreover you can enter:
--- sqrt( ) for the square root, abs( ) for the absolute value;
--- max(,) for the maximim of two entries, min(,) for the minimum of two entries, e.g. max(sin(2*x),0)
--- floor( ), ceil( ), round( ) for the previous, next, and the nearest integer, respectively.

The range boxes for x and y accept numerical entries and mulpiplies of pi, for example pi/2, 2*pi.
After you enter formulas for one or more functions and the x-, y-ranges, click the GRAPH button. Whenever you change x-, y- ranges or one of your function formula, click the GRAPH button for the changes to take effect.

You can draw on the board using the mouse. Click ERASE to erase your drawing.

Powers associate to the right. For example: 2^3^4=2^(3^4). sin(x+1)^2 stands for (sin(x+1))^2. When in doubt, use parentheses.

Some examples:

for $f(x) = 2x - x^2$ enter **2 * x - x ^ 2**

for $f(x) = \sqrt{5+x}$ enter **sqrt(5 + x)**

for $f(x) = 7\cos(3x)$ enter **7 * cos(3 * x)**

for $f(x) = x\sin^2(x)$ enter **x * sin(x) ^ 2**

for $f(x) = -e^x$ enter enter **- e ^ x**

- Once you have entered the domain values and the expression for f(x), click **GRAPH** to display the curve.
- If you make any mistakes in the domain or expression for f(x) you can changes the respective boxes and click **GRAPH** again. If you want to start over or on a new problem click **RESET**.
- When you are satisfied with the graph shown click **SELECT POINTS**. Then go to the graph of the curve and with your mouse click on the curve to select points. A dot will appear and in the trapezoidal and Simpson's program vertical bars are drawn to establish regions. You need not choose points in order from left to right. The program will determine the order in which to use your selections.
- Once you are finished selecting points click **FINISHED SELECTING** and the program will display the corresponding approximation. To start a new approximation, click **RESET**.

EXERCISES

Directions: for each part of an exercise that involves using the routines, **print the final screen from the program.** Click on **File** at the top left of the screen and on the drop down menu click **Print**. Your name will appear on the print out. Include these print outs with any portions that were to be done by hand.

1. For $f(x) = 2x - x^2$ over [0, 2] do the following.

(a) Compute the area between f(x) and the x-axis by hand. (This will be the "exact" area.)

(b) Get a very good estimate the area between f(x) and the x-axis by using a trapezoidal approximation with at most 11 points in the partition. On your print out compute the error in your approximation.

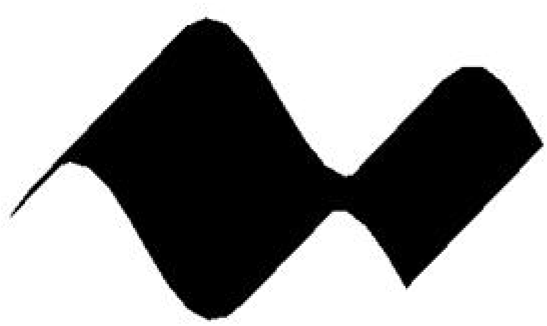
(c) Get a very good estimate the area between f(x) and the x-axis by using a Simpson's approximation using the fewest number points that you can. On your print out compute the error in your approximation.

(d) Explain why the error from part (c) is (quite) small.

2. Estimate the length of one arch of y = sin(x) by approximating $L = \int_0^{\pi} \sqrt{1 + \cos^2(x)}\,dx$ using the Simpson program where you choose 9 points that need not be equispaced.

3. Estimate the length of one arch of the curve y = sin(x) using the arc length program. Choose at most 12 points and try to get a very good estimate this way.

4. A metal fabrication company makes corrugated sheets that conform to the curve $y = \sin\left(\frac{3\pi}{20}x\right)$ for x over [0, 20]. If the sheet is to be stamped from flat sheets by a process that does not stretch the material, how wide should the original material be? Approximate the arclength of the corrugated sheet using either the Trapezoidal or Simpson program. (Adapted from University Calculus, by Hass, Weir, and Thomas, published by Pearson Education, 2005)



Corrugated sheet.

5. Estimate the length of the curve given in Exercise 4 using the arc length approximation program.